MATH 2010B Advanced Calculus I, 2014-15
QUIZ 3

Honesty in Academic Work: *The Chinese University of Hong Kong places very high importance on honesty in academic work submitted by students, and adopts a policy of zero tolerance on cheating and plagiarism. Any related offence will lead to disciplinary action including termination of studies at the University.*

NAME:_____ ID:_____

Instruction: Answer ALL questions and show your work with explanation.

Question 1: Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by

$$f(x,y) = \begin{cases} \dfrac{2xy^2}{x^2+y^4} & \text{when } (x,y) \neq (0,0), \\ 0 & \text{when } (x,y) = (0,0). \end{cases}$$

(a) (5 points) Compute the directional derivative $D_{\mathbf{u}} f(0,0)$ for the direction $\mathbf{u} = (\cos\theta, \sin\theta)$ at the origin $(0,0)$.

Answer:

(b) (5 points) Is f differentiable at $(0,0)$? Explain clearly your answer.

Answer:

(c) (6 points) Find all the points $(x_0, y_0) \in \mathbb{R}^2$ where $\nabla f(x_0, y_0) = \mathbf{0}$. Sketch the set

$$S = \{(x, y) \in \mathbb{R}^2 \ : \ \nabla f(x_0, y_0) = \mathbf{0}\}.$$

Answer:

(d) (4 points) Let $\gamma(t) = (\cos^2 t, (1+t)^{100})$, $t \in \mathbb{R}$. Compute the derivative $\left.\dfrac{d}{dt}\right|_{t=0} f(\gamma(t))$.

Answer:

——End of Quiz 3——